SVKM's NMIMS
MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING

Programme: B.Tech/ MBA Tech (IT) Year: II Semester: III

Academic Year: 2019-20

Subject: Engineering Mathematics – III

Date: 05 November 2019

Marks: 100
Time: 2.00 pm - 5.00 pm
Durations: 3 (Hrs)
No. of Pages: 8

Final Examination (2019-20)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. 1 is compulsory.
2) Out of remaining questions, attempt any 4 questions.
3) **In all 5 questions to be attempted.**
4) All questions carry equal marks.
5) **Answer to each new question to be started on a fresh page.**
6) **Figures in brackets on the right hand side indicate full marks.**
7) **Assume suitable data if necessary.**

Q.1 a) **i)** Which central moments is known as Variance **(1)**

a) μ_2 b) μ_3 c) μ_2' d) None of these

ii) Let X be a continuous random variable with probability distribution **(1)**

$$f(x)=\begin{cases}\frac{x}{6}+k & 0\le x\le 3\\ 0 & \text{otherwise}\end{cases}$$ then the value of k is

a) $\frac{-1}{12}$ b) $\frac{7}{12}$ c) $\frac{1}{12}$ d) None of these

iii) The joint probability distribution of X and Y is given by **(1)**

$$f(x,y)=\begin{cases}15e^{-3x-5y} & x>0,y>0\\ 0 & \text{elsewhere}\end{cases}$$ then marginal distribution of X is

a) $5e^{-5y}$ b) $3e^{-3x}$ c) $3e^{-5y}$ d) None of these

iv) If a random a random variable X follows Poisson distribution such that $P(X=1)=4P(X=2)$ then mean of the distribution is **(1)**

a) 2 b) 1 c) 1/2 d) None of these

v) Write down the normal equations of the line $y=a+bx$ **(1)**

vi) From a pack of cards two cards are drawn. What is the probability that both are black. **(1)**

a) $\frac{25}{78}$ b) $\frac{1}{17}$ c) $\frac{13}{20}$ d) $\frac{25}{102}$

vii) Mean of exponential distribution is **(1)**

a) λ b) $\frac{1}{\lambda^2}$ c) $\frac{1}{\lambda}$ d) None of these

viii) Conditional function of Y given X i.e. $f(y|x)$ is **(1)**

a) $\frac{f(x,y)}{f_X(x)}$ b) $\frac{f(x,y)}{f_Y(y)}$ c) $\frac{f_Y(y)}{f(x,y)}$ d) $\frac{f_X(x)}{f(x,y)}$

ix) If the two lines of regression are $x+3y=5$ and $4x+3y=8$ then the coefficient of correlation is 0.5 **(1)**

a) False b) True

x) When sample size is greater than 30 to compare means, we use **(1)**

a) F-test b) t-test c) Z-test d) χ^2-test

b) Match the following **(10)**

A)	A bag contains 8 white and 6 red balls. What is the probability of drawing 2 balls of same colour	i) $\frac{7}{27}$
B)	If $f(x)=\begin{cases}\frac{x^2}{9}, & 0<x<3 \\ 0, & \text{otherwise}\end{cases}$ then $P(1<x<2)$ is	ii) $\frac{43}{91}$
C)	The joint probability distribution is given by $P(X=x,Y=y)=k(4x+3y);\ x=1,2;y=1,2$ then the value of k is	iii) $\frac{1}{4}$
D)	If mean of Binomial distribution is 4 and variance is 3 then value of p is	iv) $\frac{1}{42}$
E)	Equation of line x on y is	v) $x-\bar{x}=bxy\left(y-\bar{y}\right)$
		vi) $y-\bar{y}=byx\left(x-\bar{x}\right)$

(6)

Q.2 **(a)** From a group of 3 Indians, 4 Sri Lankans and 5 Americans, a sub-committee of 4 peoples is selected by lots. Find the probability that the sub-committee will consist of i) 2 Indians and 2 Sri Lankans ii) 1 Indian, 1 Sri Lankan and 2 Americans iii) 4 Americans

(b) There are 40 girls and 20 boys in a class. 8 students of this class of which 3 were girls formed a Violin club. (i) Given that a student of this class does not play violin, what is the probability that the student is a boy? (ii) Given that the student is girl, what is the probability she plays violin? **(6)**

(c) A random variable X has following probability function (8)

X	0	1	2	3	4	5	6
$P(X=x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find k, P(X < 4) and P(3 < X ≤ 6). Also find Mean and Variance

Q.3 (a) Between the hrs of 2 to 4 PM the average no. of phone calls per minute is coming into switch board of a company is 2.5. Find probability that during a particular minute there will be i) no phone call ii) 4 or less calls iii) more than 6 calls. (6)

(b) The first four moments of the distribution about the value 5 are 2, 20, 40 and 50. Find the mean and the first four central moments. Also find the coefficient of skewness and coefficient of kurtosis. (6)

(c) Marks obtained by 10 students in Economics and Statistics are given below (8)

Marks in Economics (X)	25	28	35	32	31	36	29	38	34	32
Marks in Statistics (Y)	43	46	49	41	36	32	31	30	33	39

Find i) coefficient of correlation ii) The regression Y on X iii) Estimate the marks in Statistics when the marks in Economics is 30.

Q.4 (a) A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample from a batch of a string having strength of 15.6 pounds and standard deviation of 2.2 pounds. (6)

(b) In a random sample of 500 persons from Maharashtra 250 are found to be consumers of vegetable oil. In another sample of 400 persons from Gujarat, 220 are found to be consumers of vegetable oil. Discuss whether the data reveal a significant difference. (6)

(c) Fit a second degree curve to the following data. Also find y at $x = 6.5$ (8)

x	0	1	2	3	4	5	6	7	8
y	12.0	10.5	10.0	8.0	7.0	8.0	7.5	8.5	9.0

Q.5 (a) 300 digits were chosen at random from a table of random numbers. The frequency of the digits was as follows: (6)

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	28	29	33	31	26	35	32	30	31	25

Using χ^2-test examine the hypothesis that the digits were distributed in equal numbers in the table.

(b) Two samples are drawn from two normal populations. From the following data test whether the two samples have same variance at 2% level. (6)

Sample 1	60	65	71	74	76	82	85	87		
Sample 2	61	66	67	85	78	63	85	86	88	91

(c) The marks obtained by a group of 9 regular course students and another group of 11 (8)

part time students in a test are given below:

Regular	56	62	63	54	60	51	67	69	58		
Part time	62	70	71	62	60	56	75	64	72	68	66

Examine whether the marks obtained by regular students and part time students differ significantly at 5% level.

Q.6 **(a)** The joint probability density function of random variable is given by **(6)**

$$f(x,y)=\begin{cases}\frac{1}{8}(x+y), & 0<x<2;\ 0<y<2\\ 0 & \text{otherwise}\end{cases}$$ Find i) Marginal density function of X and Y. ii) Are X and Y independent? iii) Find $f\left(\frac{y}{x}\right)$ and $f\left(\frac{x}{y}\right)$

(b) One factory F_1 produces 1000 articles, 20 of them being defective products, second factory F_2 produces 4000 articles, 40 of them are defective and third factory F_3 produces 5000 articles 50 of them being defective. All these articles are put in one stock pile. One of them is chosen and is found to be defective. What is the probability that it is from factory F_1? **(6)**

(c) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights i) greater than 72 inches ii) less than 62 inches iii) between 65 and 71 inches. **(8)**

Q.7 **(a)** a) A random variable X has following pdf **(6)**

X	-1	1	3	5
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

By using Chebyshev's Inequality prove that $P\{|X-3|\geq 1\}\leq\frac{16}{3}$

(b) An experiment of drowning a random card from an ordinary playing cards deck is done with replacement it back. This was done 10 times. Find the probability getting 2 spades, 3 diamonds, 3 club and 2 hearts. **(6)**

(c) A garage has recorded the following faults in cars of four makes over a period of time. **(8)**

	Nature of fault		
Make	Mech	Electrical	Chasis
A	32	26	12
B	60	54	22
C	52	38	16
D	28	25	12

Using χ^2-test can you say the type of fault is independent of the make of the car?

TABLE A.2

t Distribution: Critical Values of *t*

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

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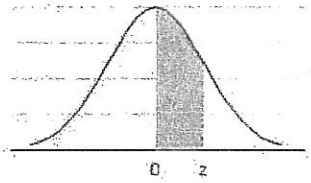
Table of Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

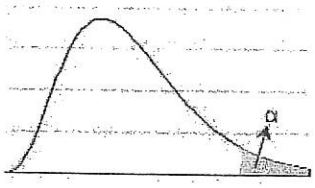
Note:

Middle 50% area = ± 0.6745
Middle 90% area = ± 1.6449
Middle 95% area = ± 1.9600
Middle 99% area = ± 2.5758

For any Normal Variate (with Mean = μ and Std. Deviation = σ):

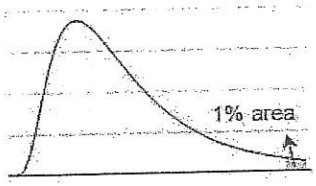
μ ± 1σ = 0.6827
μ ± 2σ = 0.9545
μ ± 3σ = 0.9973

Table of Chi-Square (χ^2) Distribution



	← Level of significance (α) →									
d.f. ↓	**0.995**	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2103	10.5966
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8382
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8603
5	0.4117	0.5543	0.8312	1.1455	1.6103	9.2364	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5476
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9550
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5894
10	2.1559	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1882
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6751	21.9200	24.7250	26.7568
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2995
13	3.5650	4.1069	5.0088	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8195
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3193
15	4.6009	5.2293	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5779	32.8013
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.6972	6.4078	7.5642	8.6718	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.2648	7.0149	8.2307	9.3905	10.8649	25.9894	28.8693	31.5264	34.8053	37.1565
19	6.8440	7.6327	8.9065	10.1170	11.6509	27.2036	30.1435	32.8523	36.1909	38.5823
20	7.4338	8.2604	9.5908	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.0337	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322	41.4011
22	8.6427	9.5425	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7957
23	9.2604	10.1957	11.6886	13.0905	14.8480	32.0069	35.1725	38.0756	41.6384	44.1813
24	9.8862	10.8564	12.4012	13.8484	15.6587	33.1962	36.4150	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9279
26	11.1602	12.1981	13.8439	15.3792	17.2919	35.5632	38.8851	41.9232	45.6417	48.2899
27	11.8076	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9629	49.6449
28	12.4613	13.5647	15.3079	16.9279	18.9392	37.9159	41.3371	44.4608	48.2782	50.9934
29	13.1211	14.2565	16.0471	17.7084	19.7677	39.0875	42.5570	45.7223	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4927	20.5992	40.2560	43.7730	46.9792	50.8922	53.6720

Table of 'F' Distribution. (Value of $F_{0.01}$)



← Degrees of Freedom for the Denominator →	← Degrees of Freedom for the Numerator →								
	1	2	3	4	5	6	7	8	9
1	4,052.18	4,999.50	5,403.35	5,624.58	5,763.65	5,858.99	5,928.36	5,981.07	6,022.47
2	98.5025	99.0000	99.1662	99.2494	99.2993	99.3326	99.3564	99.3742	99.3881
3	34.1162	30.8165	29.4567	28.7099	28.2371	27.9107	27.6717	27.4892	27.3452
4	21.1977	18.0000	16.6944	15.9770	15.5219	15.2069	14.9758	14.7989	14.6591
5	16.2582	13.2739	12.0600	11.3919	10.9670	10.6723	10.4555	10.2893	10.1578
6	13.7450	10.9248	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7	12.2464	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8	11.2586	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.5614	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.0443	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
30	7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120	6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6433	4.6122	3.7881	3.3254	3.0233	2.8080	2.6453	2.5172	2.4132